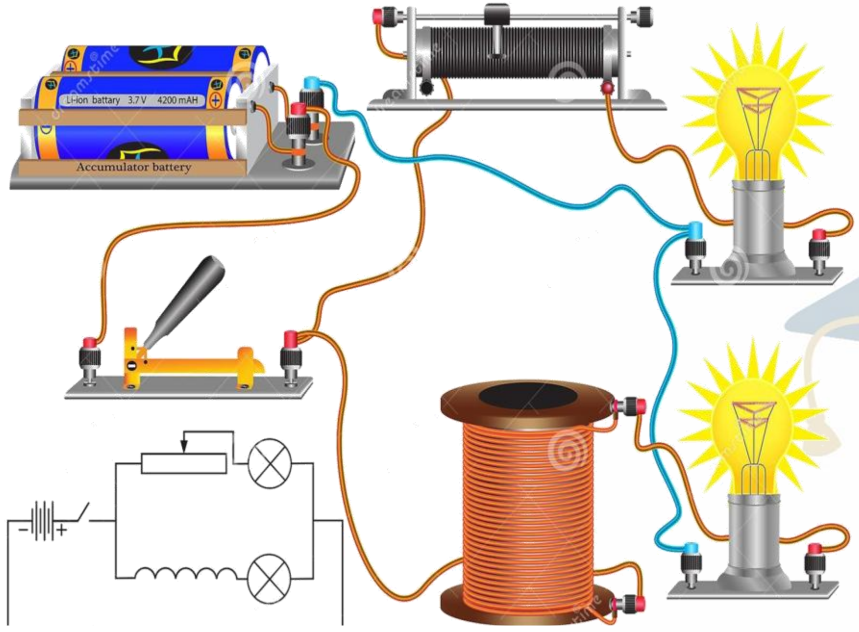
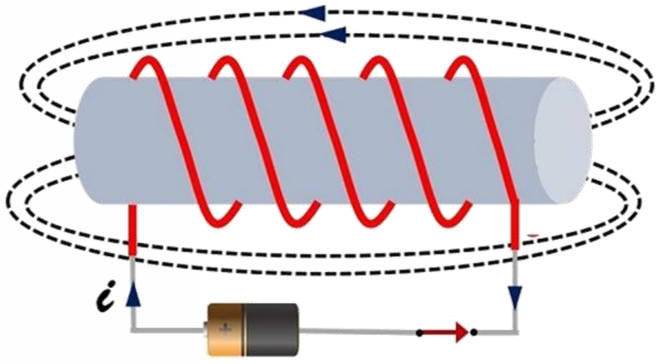


What is Self Induction?

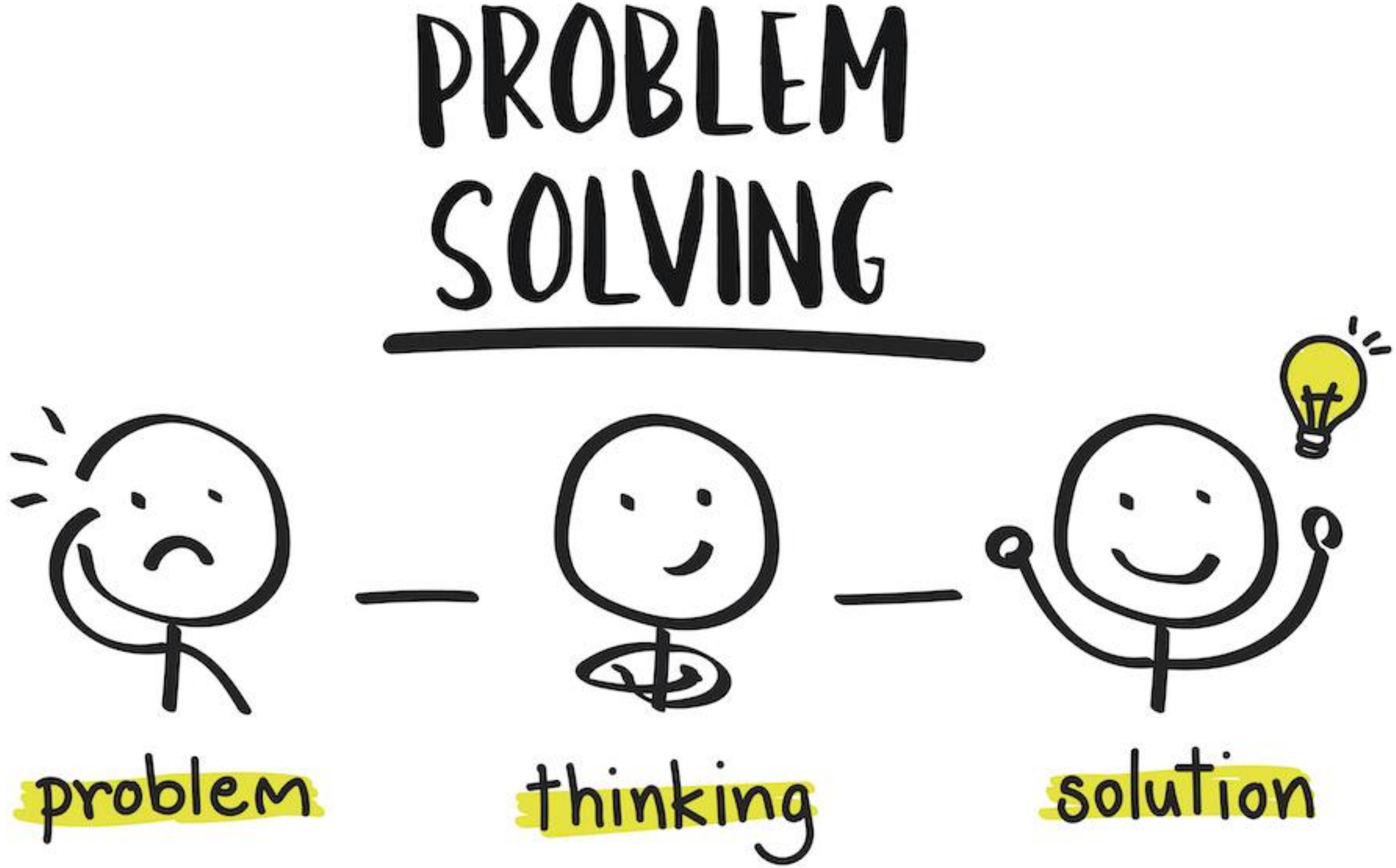


Physics – Grade 12

Unit Two – Electricity

Chapter 9 – Self Induction

Prepared & Presented by: Mr. Mohamad Seif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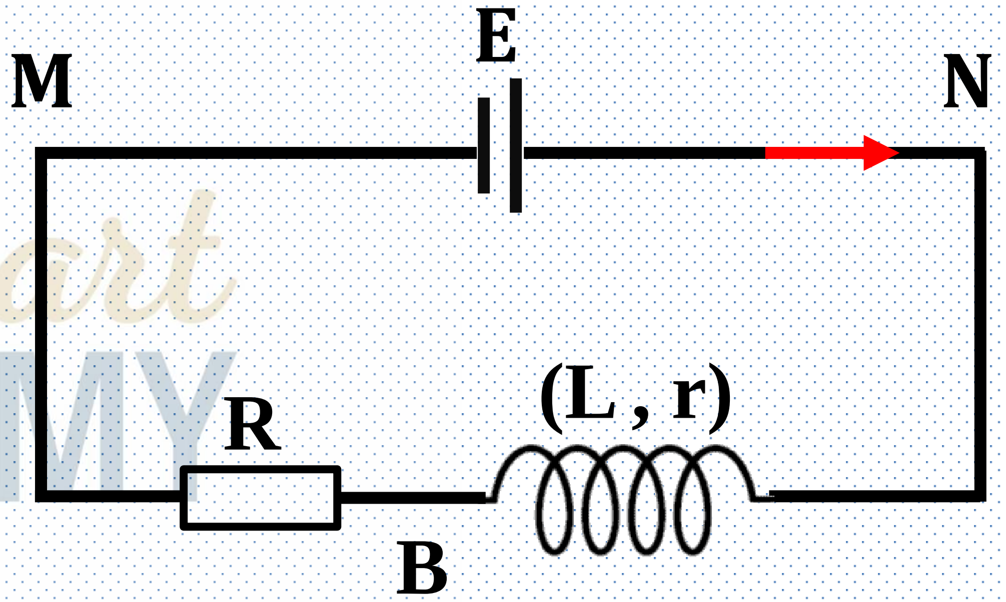
Exercise 1

To determine the characteristics of a coil, we set up the circuit of the adjacent figure which is formed of the coil (L, r), a resistor of resistance $R = 2.5\Omega$, a generator of constant emf $E = 6\,V$ and of negligible resistance and a switch K.

At the instant $t_0 = 0$, we close the switch K.

At an instant t, the circuit carries a current i.



Exercise 1

1) Derive the differential equation describing the variations of i as a function, of time.

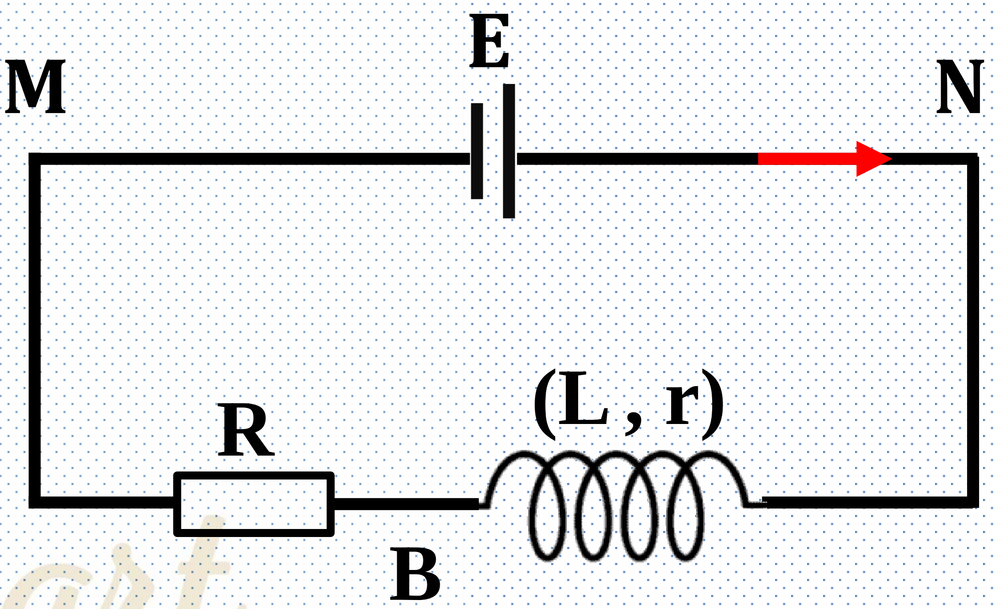
2) The solution of this equation is of the form $i = I_0(1 - e^{-\frac{t}{\tau}})$, where I_0 and τ are constants.

Determine, in terms of E, R_{eq} and L, the expressions of I_0 and τ.

3) Give, as a function of time, the expression of the voltage u_{MB}.

Exercise 1

1)Derive the differential equation describing the variations of i as a function, of time.



Apply law of addition of voltages:

$$u_{NM} = u_{NB} + u_{BM}$$

$$E = ri + L\frac{di}{dt} + Ri$$

$$E = (R + r)i + L\frac{di}{dt}$$

Where $R_{eq} = (R + r)$

Differential equation in terms of current i

Exercise 1

2) The solution of this equation is of the form $i = I_0(1 - e^{-\frac{t}{\tau}})$, where I_0 and τ are constants. Determine, in terms of the given, the expressions of I_0 and τ.

$$E = R_{eq}i + L\frac{di}{dt}$$

$$i = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\frac{di}{dt} = \frac{I_0}{\tau}.e^{-\frac{t}{\tau}}$$

Substitute i and $\frac{di}{dt}$ in differential equation.

$$E = R_{eq}I_0\left(1 - e^{-\frac{t}{\tau}}\right) + L.\frac{I_0}{\tau}.e^{-\frac{t}{\tau}}$$

Exercise 1

$$E = R_{eq}I_0 - R_{eq}I_0e^{-\frac{t}{\tau}} + \frac{LI_0}{\tau}.e^{-\frac{t}{\tau}}$$

$$E + R_{eq}I_0e^{-\frac{t}{\tau}} = R_{eq}I_0 + \frac{LI_0}{\tau}.e^{-\frac{t}{\tau}}$$

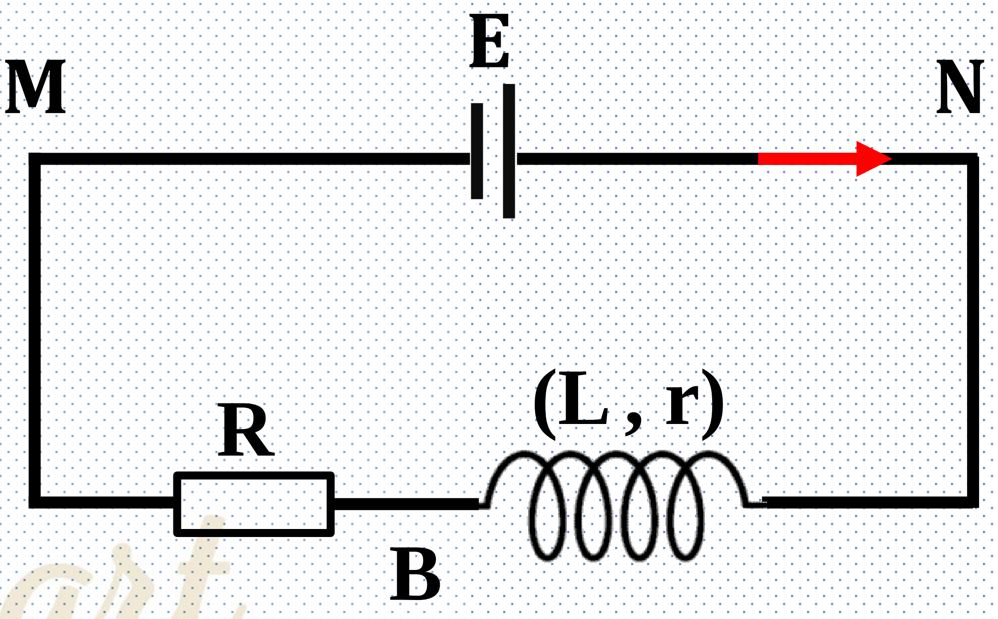
By identification we get:

$$E = R_{eq}I_0 \quad \Rightarrow \quad I_0 = \frac{E}{R_{eq}}$$

$$R_{eq}I_0 = \frac{LI_0}{\tau} \quad \Rightarrow \quad \tau = \frac{L}{R_{eq}}$$

Exercise 1

3) **Give, as a function of time, the expression of the voltage u_{MB}.**



$$u_{MB} = -Ri$$

$$u_{MB} = -R\left[I_0\left(1 - e^{-\frac{t}{\tau}}\right)\right]$$

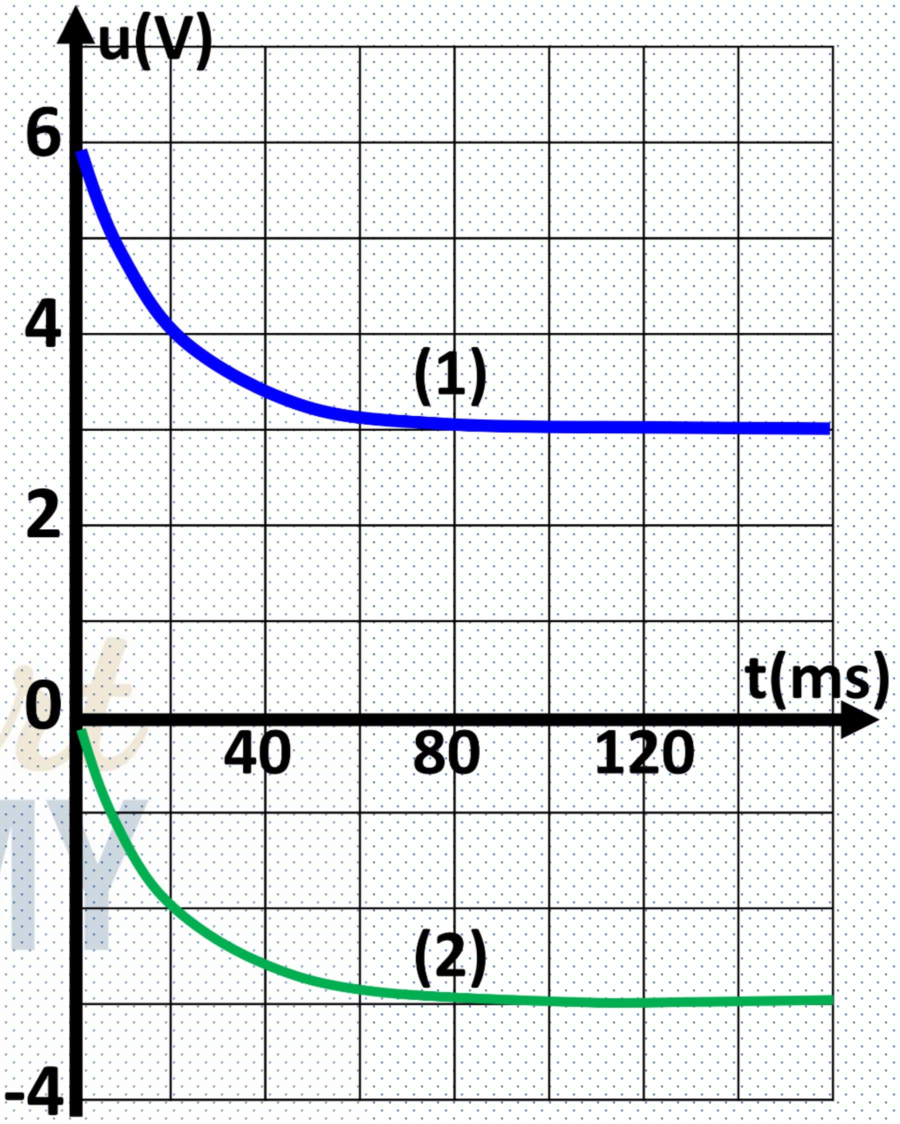
$$u_{MB} = -RI_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

Exercise 1

4) The variations of the voltages u_{NB} and u_{MB}, as a function of time, are given in the adjacent waveform.

a) Specify, with justification, the curve which gives the variations of u_{BM} as a function of time.

b) Determine the value of r and that of I_0.



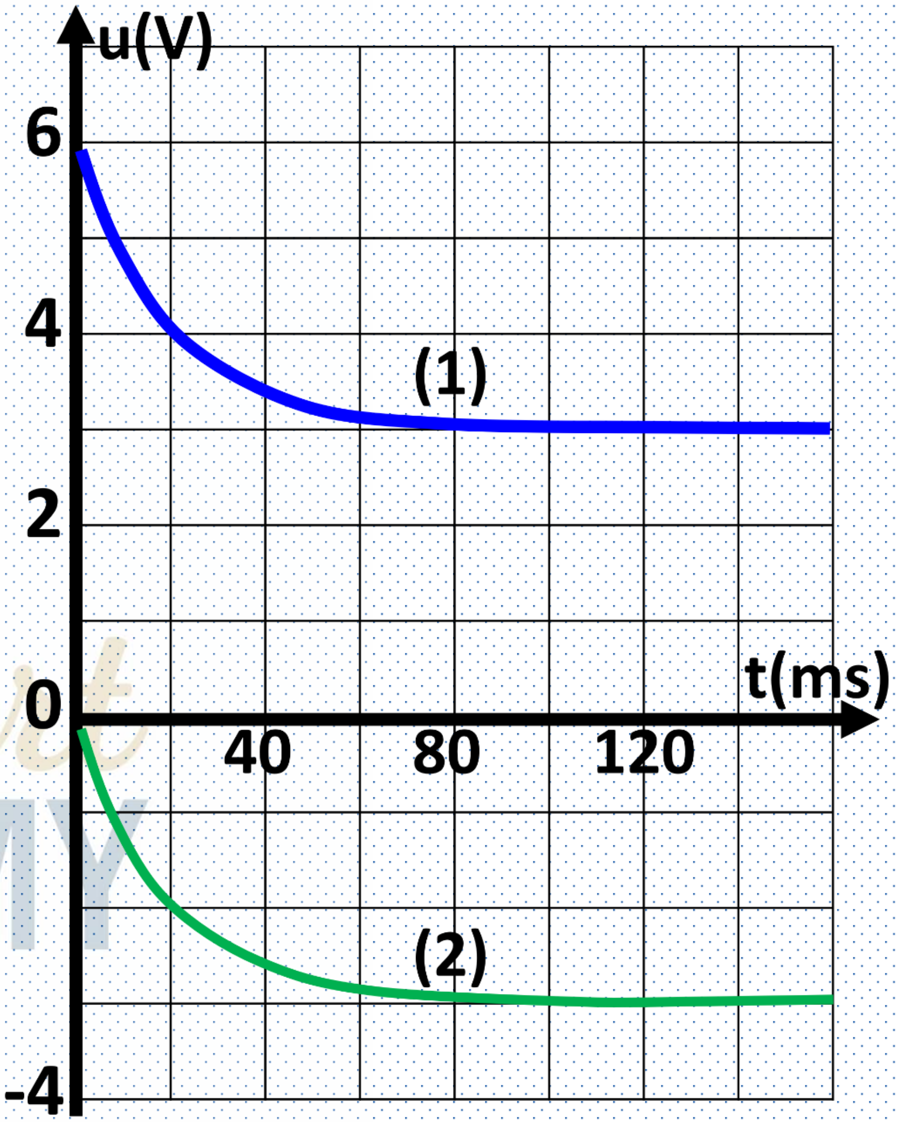
Exercise 1

a) Specify, with justification, the curve which gives the variations of u_{BM} as a function of time.

Curve (2) is for u_{MB} because according to the given circuit

$$u_{MB} = -Ri$$

So, the voltage is negative.



Exercise 1

b) Determine the value of r and that of I_0.

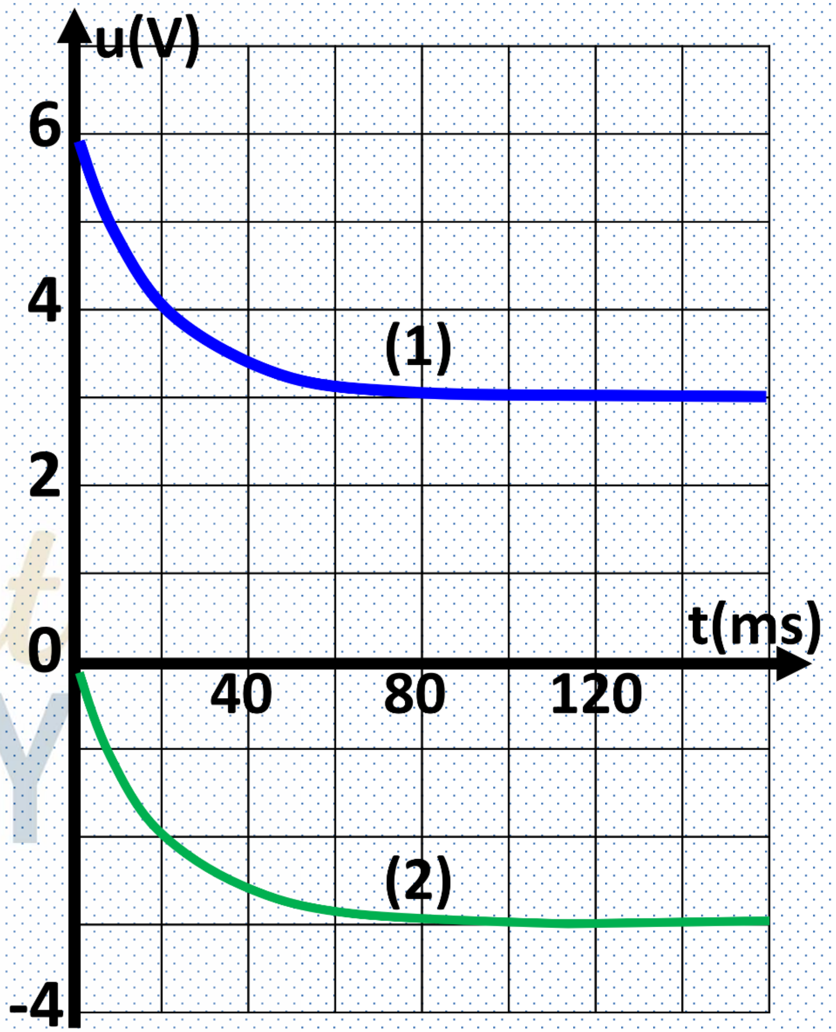
$$u_{MB} = -Ri$$

$$u_{MB(max)} = -RI_0$$

$$-3 = -2.5I_0$$

$$I_0 = \frac{-3}{-2.5}$$

$$I_0 = 1.2A$$



Exercise 1

$$I_0 = \frac{E}{R_{eq}} \Rightarrow I_0 = \frac{E}{R + r}$$

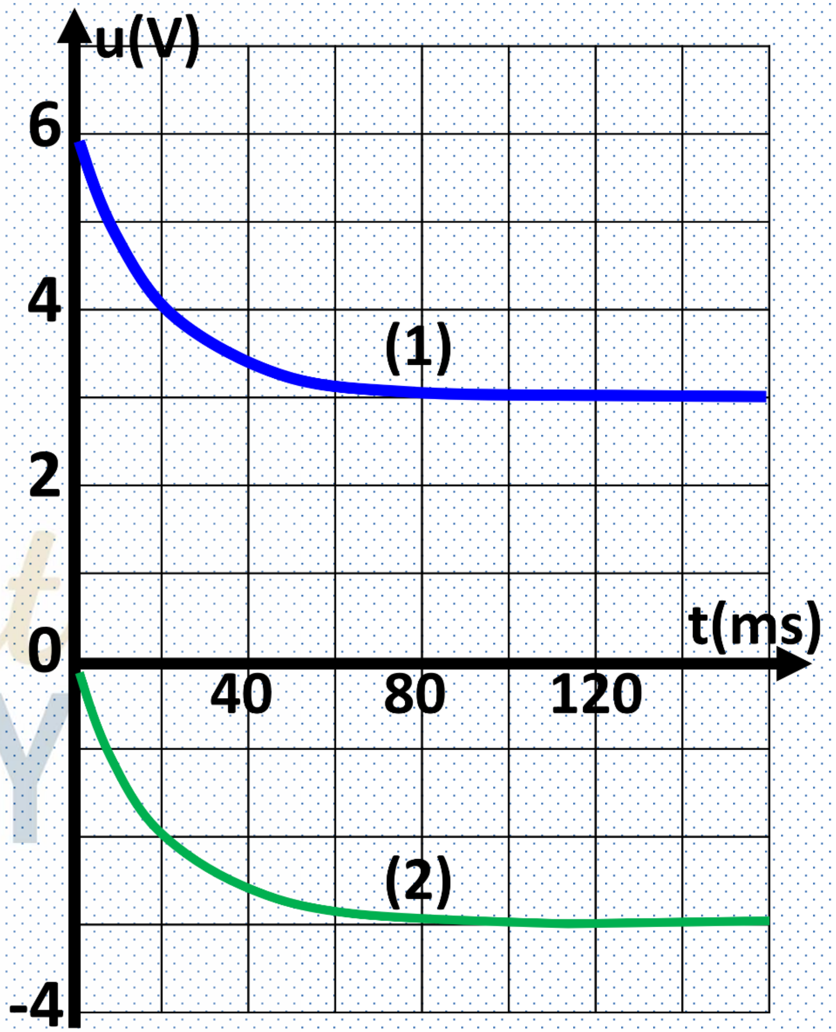
$$1.2 = \frac{6}{2.5 + r}$$

$$2.5 + r = \frac{6}{1.2}$$

$$2.5 + r = 5$$

$$r = 5 - 2.5$$

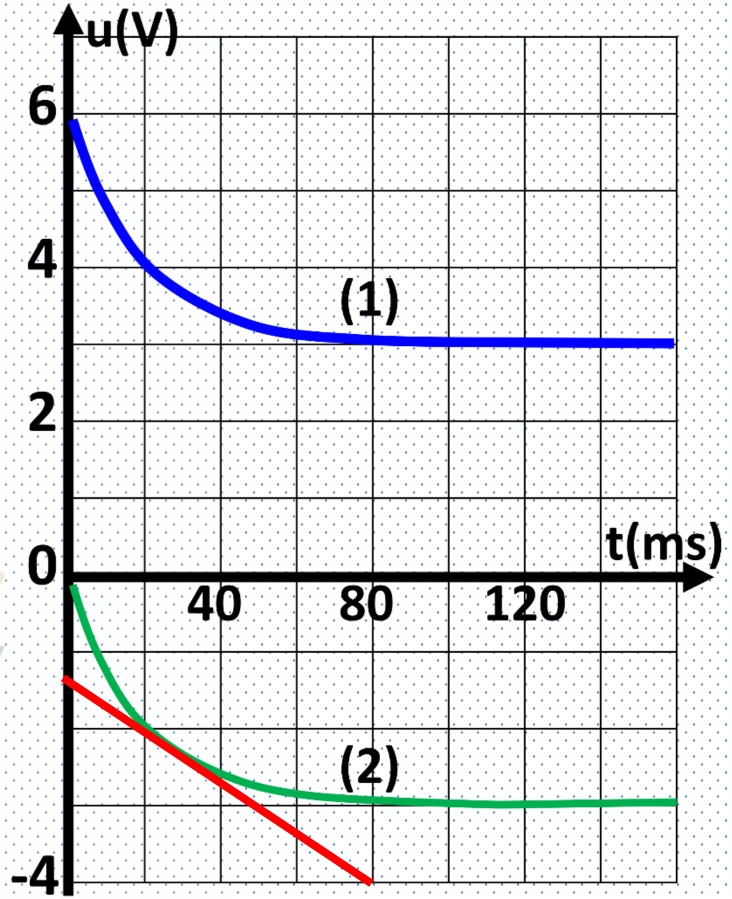
$$r = 2.5\Omega$$



Exercise 1

5) Show that the equation of the tangent to the curve (2) at a point of abscissa t' is given by:

$$u = -\frac{RI_0}{\tau}.e^{-\frac{t}{\tau}}(t - t') - RI_0(1 - e^{-\frac{t}{\tau}})$$



The slope of the tangent is given by:

$$\frac{du_{MB}}{dt} = -\frac{RI_0}{\tau}.e^{-\frac{t}{\tau}}$$

At $t = t'$

$$\frac{du_{MB}}{dt} = -\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}$$

Exercise 1

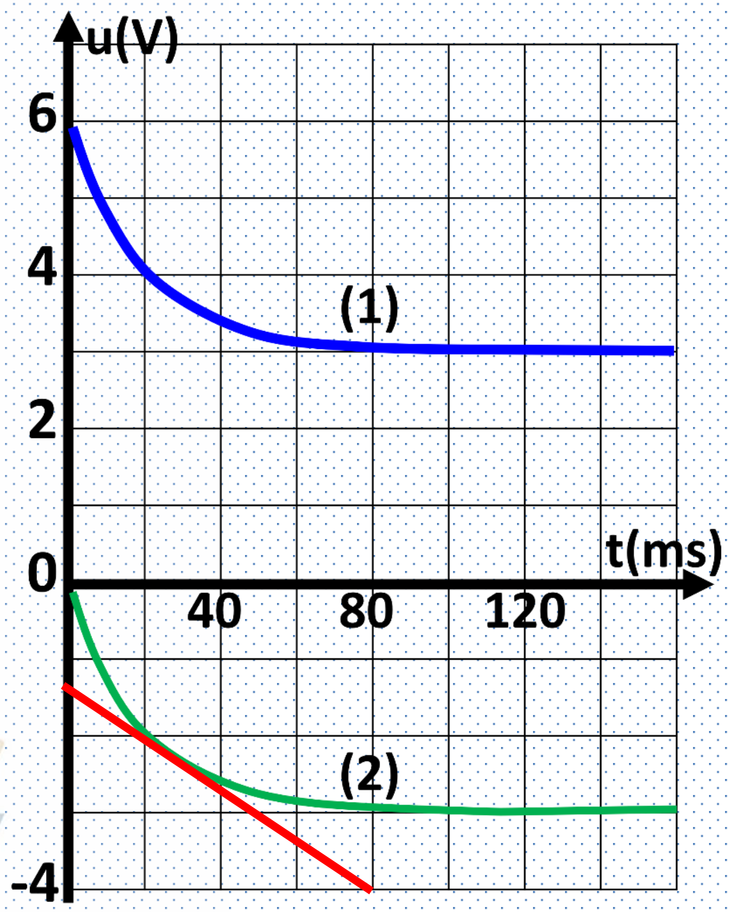
The equation of the tangent is given by:

$$u = at + b$$

$$u = -\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t + b$$

For $t = t'$ $$u = -RI_0(1 - e^{-\frac{t'}{\tau}})$$

$$-RI_0(1 - e^{-\frac{t'}{\tau}}) = -\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t' + b$$



Exercise 1

$$-RI_0\left(1-e^{-\frac{t'}{\tau}}\right)=-\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t'+b$$

$$b=\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t'-RI_0\left(1-e^{-\frac{t'}{\tau}}\right)$$

$$u=-\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t+b$$

$$u=-\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t+\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t'-RI_0\left(1-e^{-\frac{t'}{\tau}}\right)$$

Exercise 1

$$u = -\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t + \frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.t' - RI_0\left(1 - e^{-\frac{t'}{\tau}}\right)$$

$$u = -\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.[-t + t'] - RI_0\left(1 - e^{-\frac{t'}{\tau}}\right)$$

Exercise 1

6) Show that this tangent meets the asymptote to this curve at a point of abscissa $t = t' + \tau$

The asymptote is $u = -RI_0$

$$u = -\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.[-t+t'] - RI_0\left(1 - e^{-\frac{t'}{\tau}}\right)$$

$$-RI_0 = -\frac{RI_0}{\tau}.e^{-\frac{t'}{\tau}}.[-t+t'] - RI_0\left(1 - e^{-\frac{t'}{\tau}}\right)$$

$$1 = \frac{1}{\tau}.e^{-\frac{t'}{\tau}}.[-t+t'] + 1 - e^{-\frac{t'}{\tau}}$$

Exercise 1

$$1 = \frac{1}{\tau}.e^{-\frac{t'}{\tau}}.[-t+t'] + 1 - e^{-\frac{t'}{\tau}}$$

$$0 = +\frac{1}{\tau}.e^{-\frac{t'}{\tau}}.[-t+t'] - e^{-\frac{t'}{\tau}}$$

$$e^{-\frac{t'}{\tau}} = \frac{1}{\tau}.e^{-\frac{t'}{\tau}}.[-t+t']$$

$$1 = \frac{1}{\tau}.[-t+t']$$

$$\tau = [-t+t']$$

$$t' = \tau + t$$

Exercise 1

7)Deduce the values of τ and L.

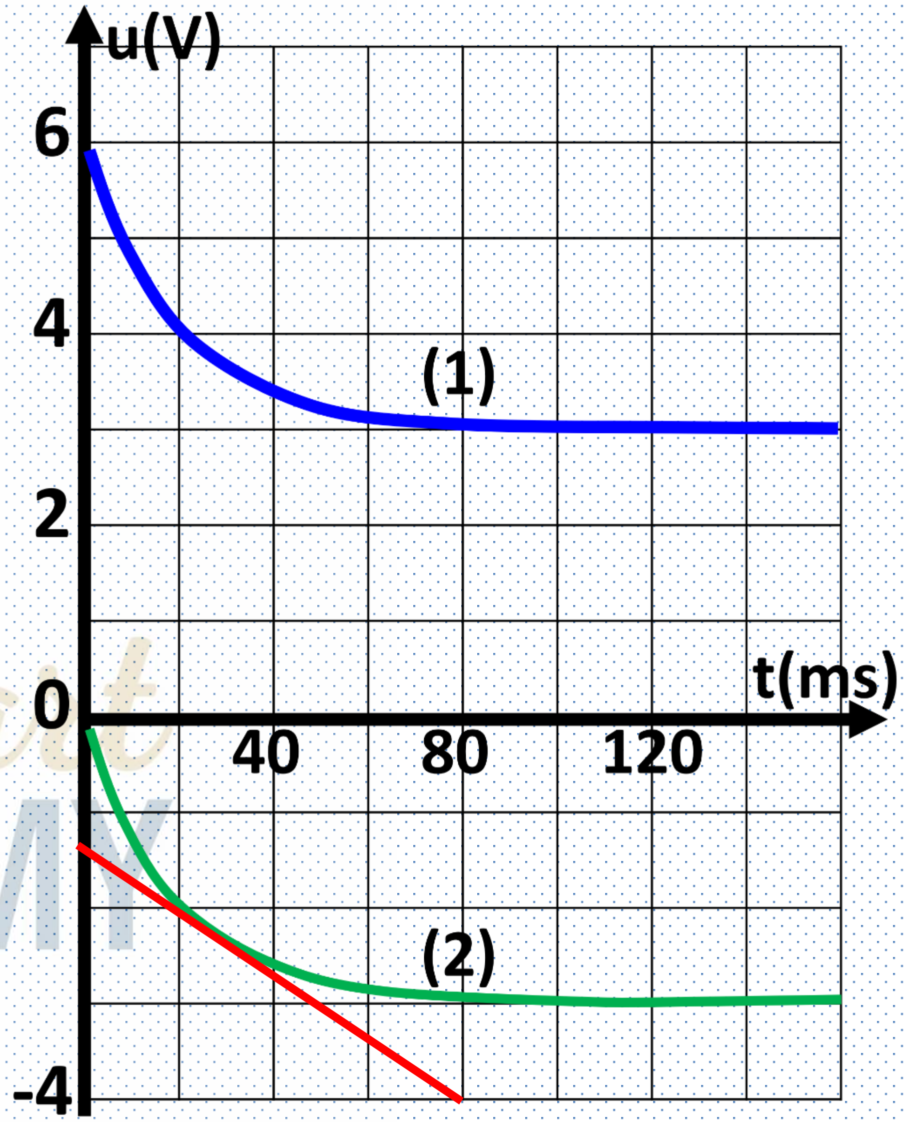
From the graph $\tau = 20ms$

$$\tau = \frac{L}{r+R}$$

$$L = \tau(R+r)$$

$$L = 20 \times 10^{-3}(2.5+2.5)$$

$$L = 0.1H$$



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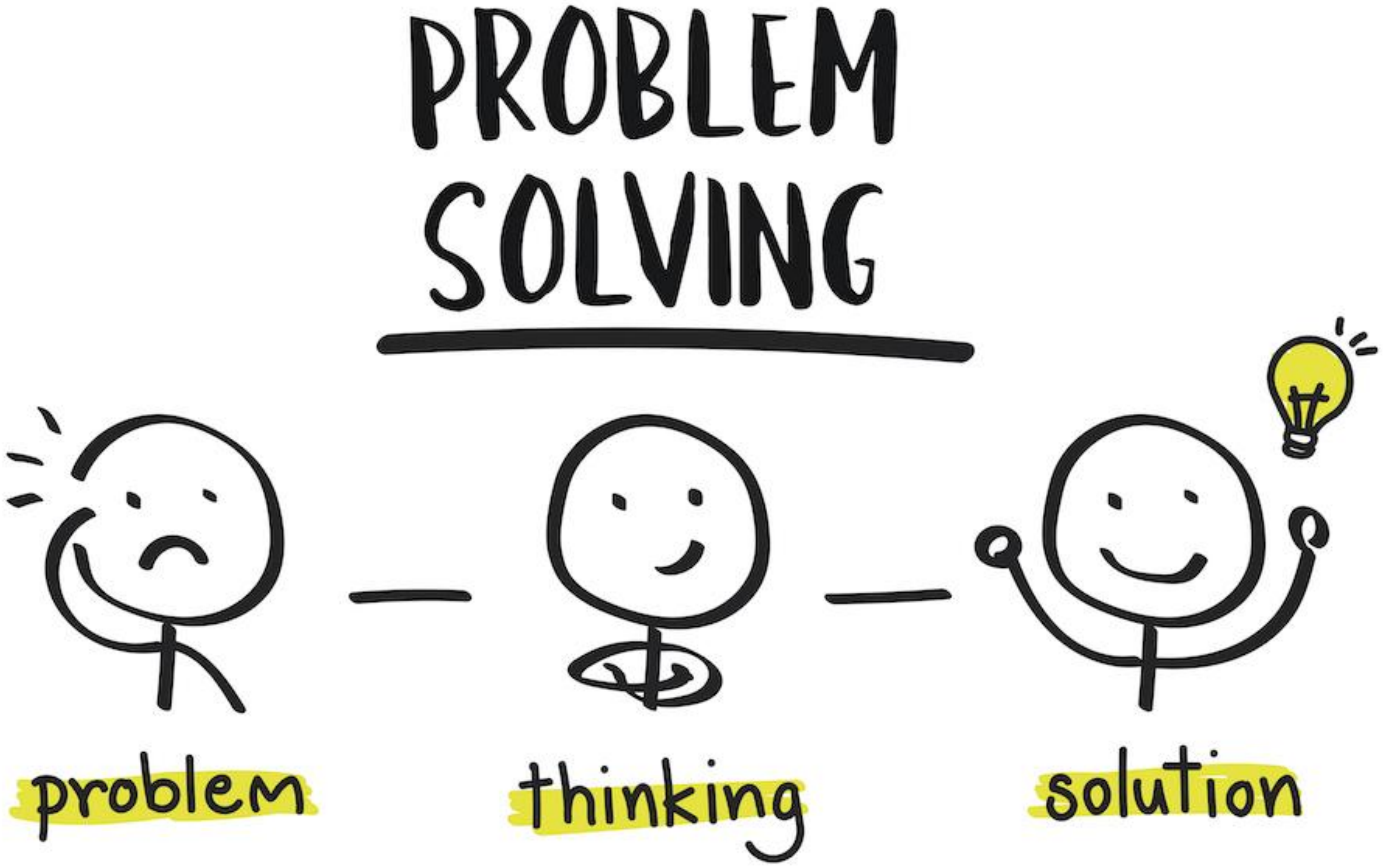
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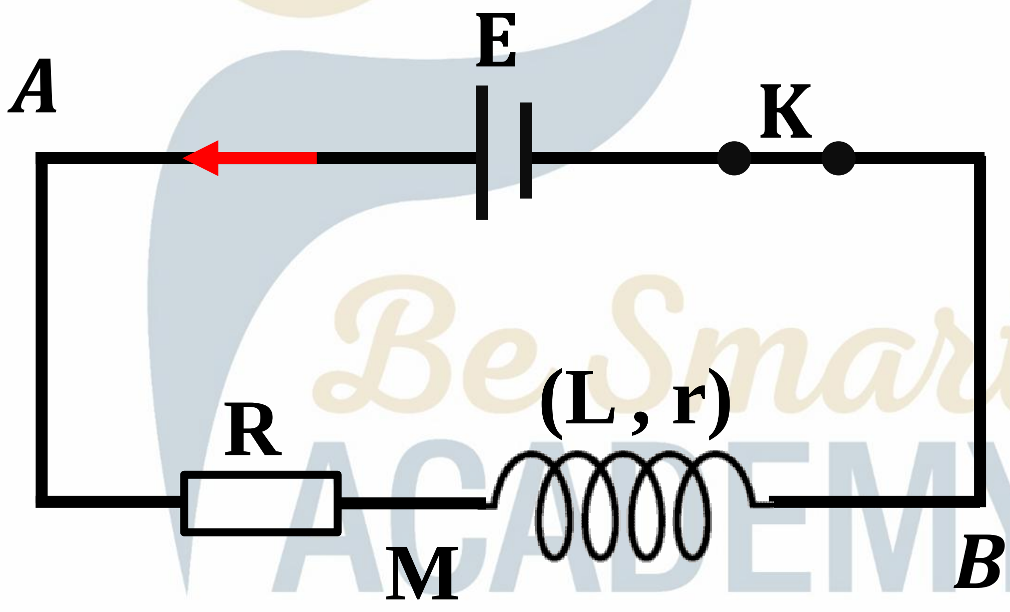
Exercise 2

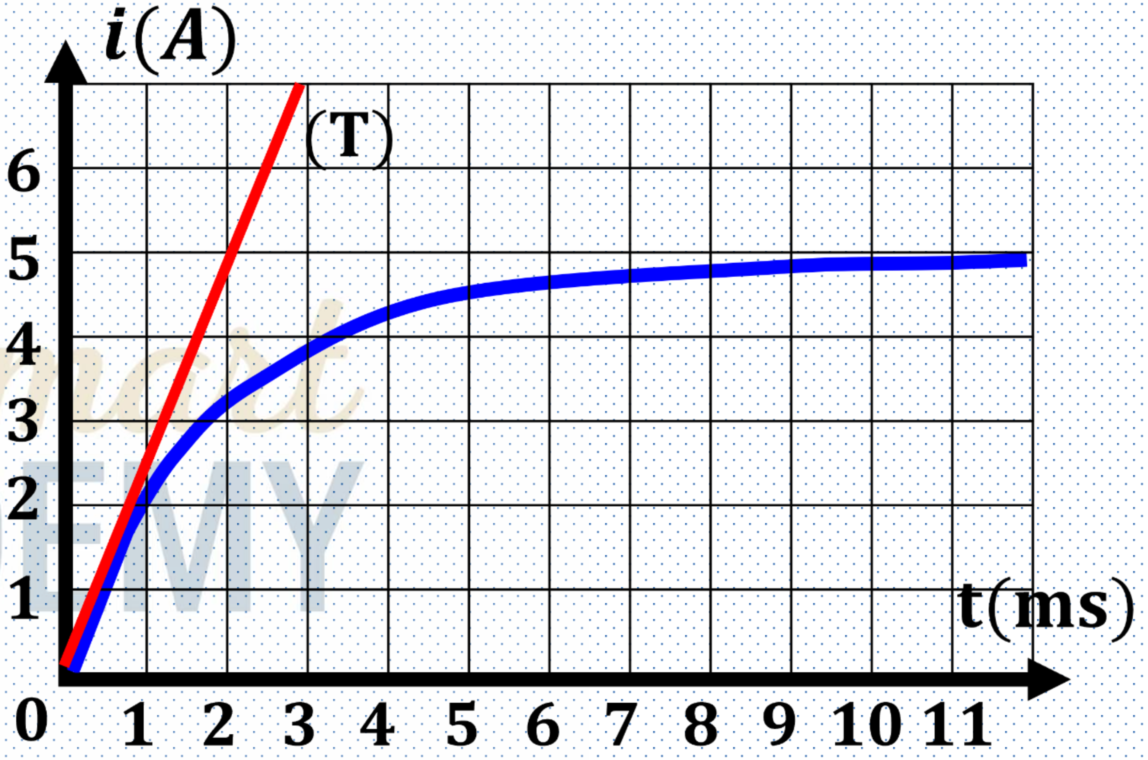
The figure below consists of an ideal generator of e.m.f E, a coil of internal resistance r & Inductance $L = 4mH$ and a resistor of resistance R.



Exercise 2

Part A: Graphical study:

The adjacent curve represents the current in the circuit as a function of time. (T) is the tangent to this curve at the point of abscissa $t_0 = 0$.

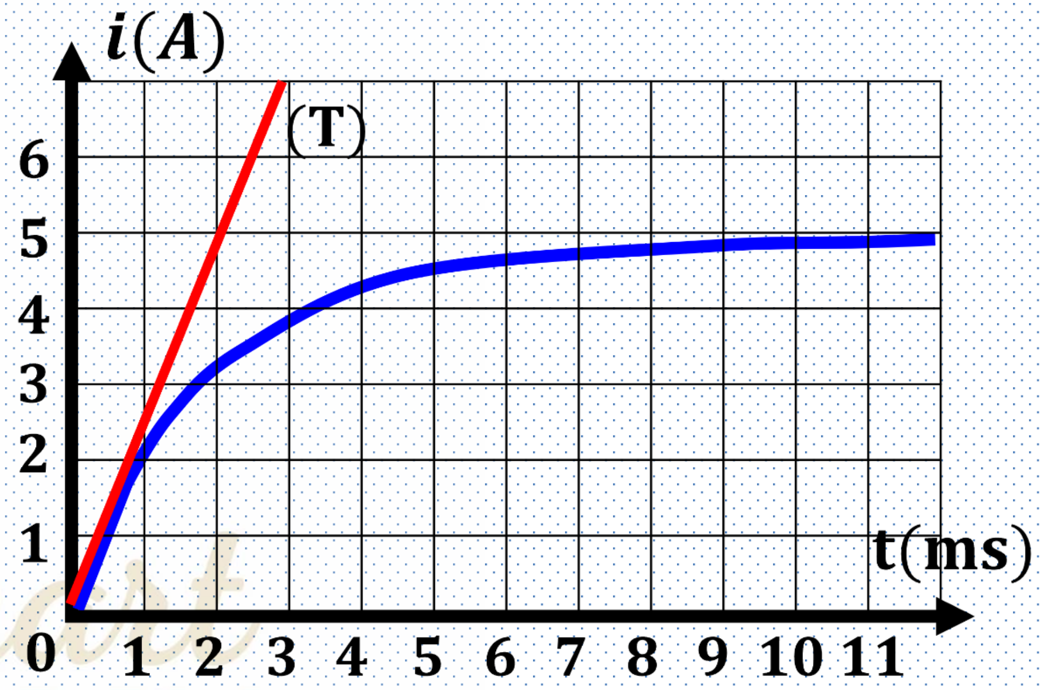


1. Justify that the coil is the seat of self-induction in $[0; 10ms]$.
2. Calculate the self-induced e.m.f e_1 at $t_0 = 0$.
3. Justify that there is no induced e.m.f. after $t = 10ms$.

Exercise 2

1. **Justify that the coil is the seat of self-induction in [0; 10ms].**

During the time interval [0; 10ms], the current is variable, so the circuit is the seat of a self-induced e.m.f "e"



2. **Calculate the self-induced e.m.f e_1 at $t_0 = 0$.**

$$e = -L\frac{di}{dt} = -4 \times 10^{-3}$$

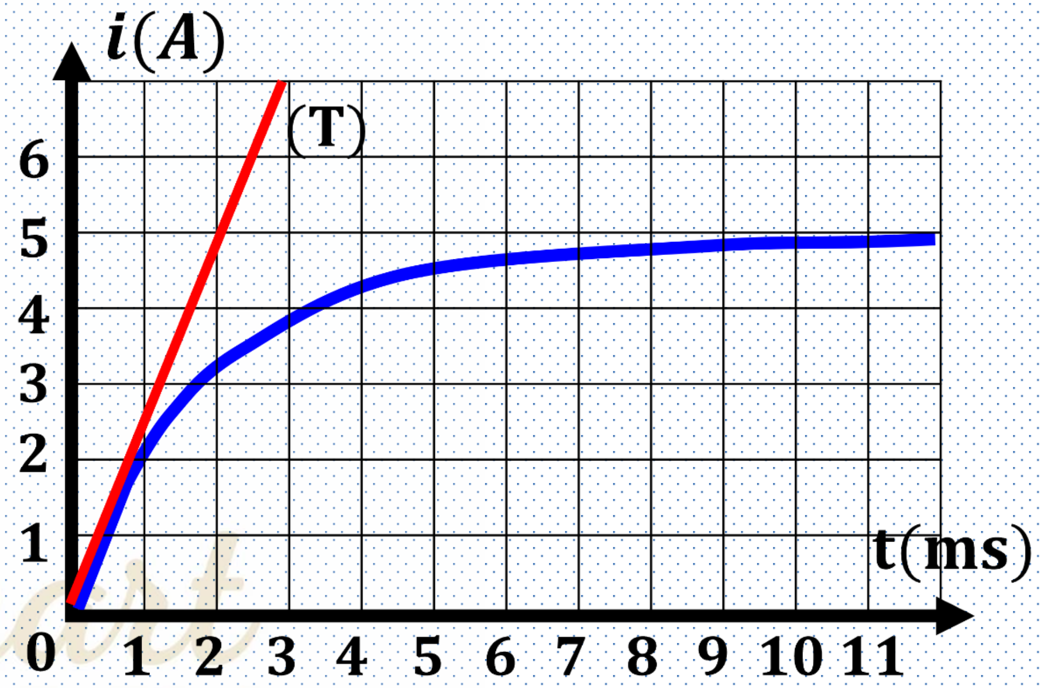
Exercise 2

2. Calculate the self-induced e.m.f e_1 at $t_0 = 0$.

$$e = -L\frac{di}{dt}$$

Where $\frac{di}{dt}$ is slope of tangent at $t = 0$

$$\frac{di}{dt} = \frac{\Delta i}{\Delta t} = \frac{5-0}{(2-0)\times 10^{-3}} = 2500A/s$$

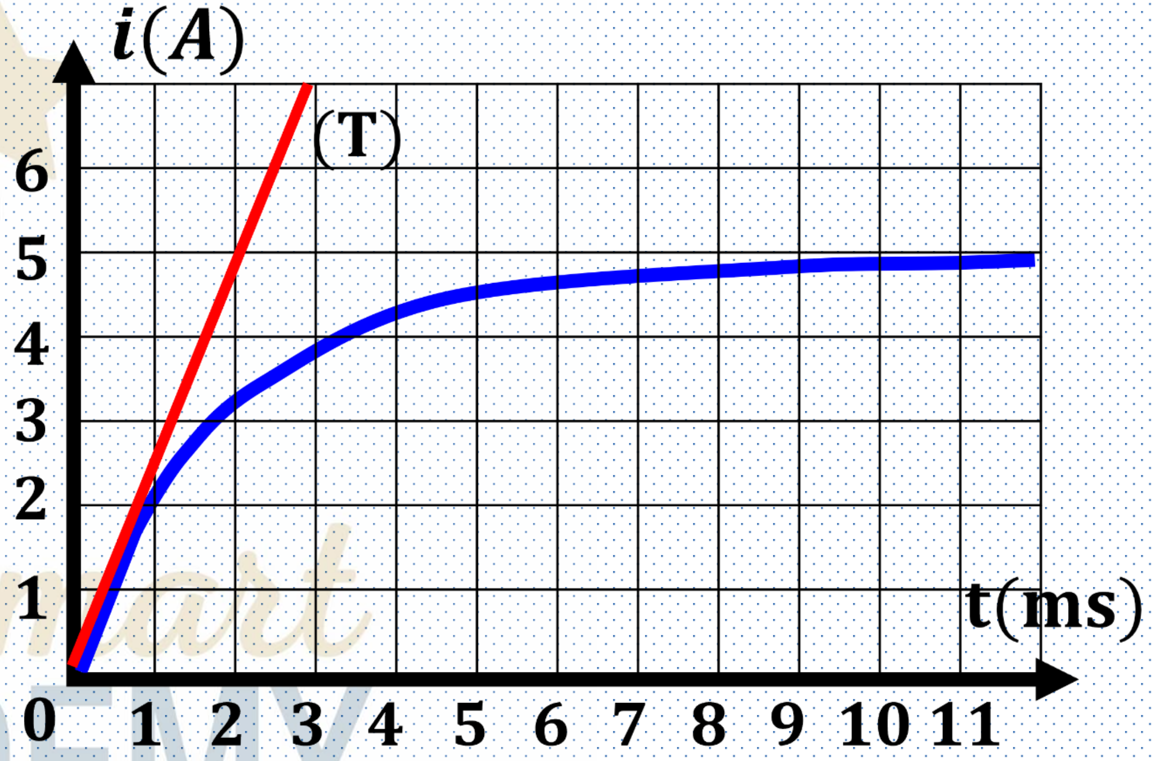


$$e = -L\frac{di}{dt} \Rightarrow e = -4\times 10^{-3}\times(2500) \Rightarrow e = -10V$$

Exercise 2

3. Justify that there is no induced e.m.f. after $t = 10ms$.

For $t > 10ms$, the steady state is reached, and the current becomes constant at maximum value.

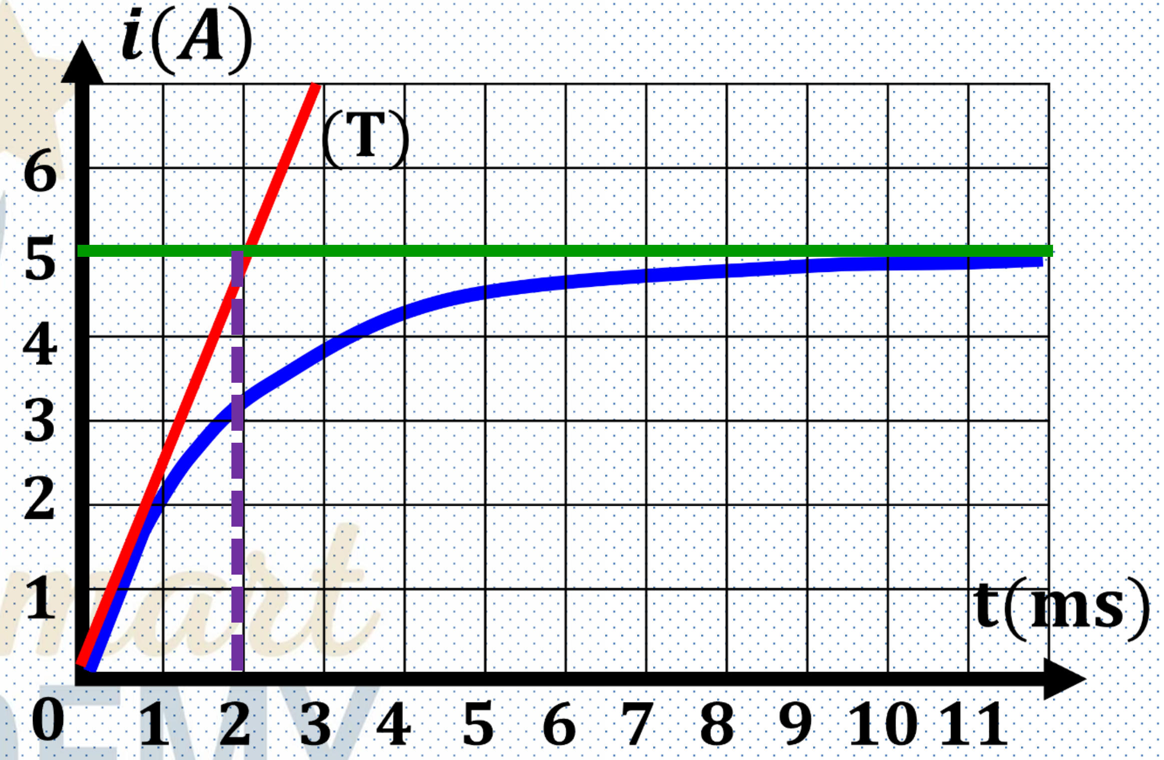


$$e = -L\frac{di}{dt} = -L(0) = 0$$

So, there is no induced e.m.f "e"

Exercise 2

4. **Using the tangent (T), determine the value of the time constant τ.**



The time constant τ is the abscissa of the point of intersection between the tangent (T) and the horizontal line $i = I_{max}$.

$$\tau = 2ms$$

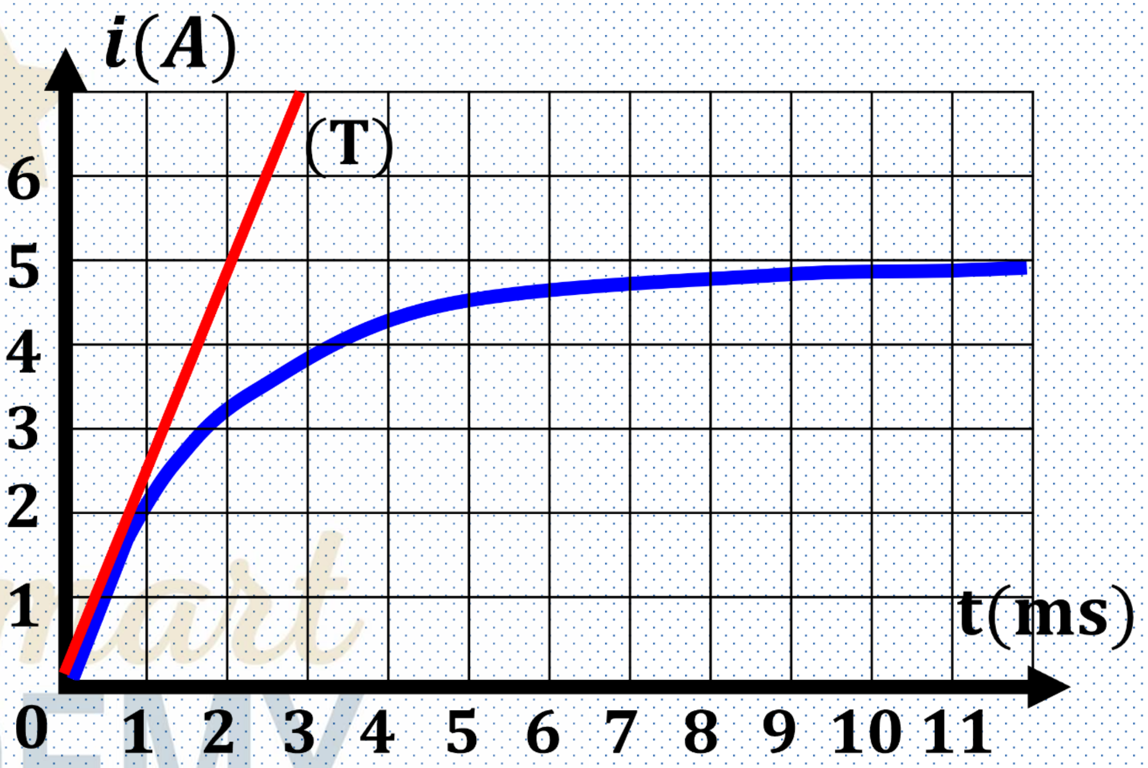
Exercise 2

5. Calculate the magnetic energy stored in the coil when the steady state is reached

$$E = \frac{1}{2} L I_0^2$$

$$E = 0.5 \times 4 \times 10^{-3} (5^2)$$

$$E = 0.05 J$$



Exercise 2

Part B: Theoretical study:

1. Give the relation between the self-induced electromotive force e_1, with L and $\frac{di}{dt}$.
2. Show that the differential equation that governs the evolution of the self-induced e.m.f. is given by:
$$\frac{de_1}{dt} + \left(\frac{R+r}{L}\right).e_1 = 0$$
3. Applying the law of addition of voltages, show that at $t_0 = 0$ the induced e.m.f. $e_1 = -E$

Exercise 2

Part B: Theoretical study:

1. **Give the relation between the self-induced electromotive force e_1, with L and $\frac{di}{dt}$.**

$$e_1 = -\frac{d\emptyset}{dt}$$

Where the magnetic flux $\emptyset$ is given by $\emptyset = Li$

$$e_1 = -\frac{d\emptyset}{dt} = -\frac{d(Li)}{dt}$$

$$e_1 = -L\frac{di}{dt}$$

Exercise 2

2. Show that the differential equation that governs the evolution of the self-induced e.m.f. is given by:

$$\frac{de_1}{dt} + \left(\frac{R+r}{L}\right).e_1 = 0$$

$$u_{AB} = u_{AM} + u_{MB}$$

$$E = Ri + (ri - e_1)$$

$$E = (R+r)i - e_1$$

Exercise 2

$$E = (R + r)i - e_1$$

Derive this equation w.r.t time:

$$0 = (R + r)\frac{di}{dt} - \frac{de_1}{dt}$$

But $e_1 = -L\frac{di}{dt}$

$$\frac{di}{dt} = -\frac{e_1}{L}$$

$$0 = (R + r)\left[-\frac{e_1}{L}\right] - \frac{de_1}{dt}$$

$$\frac{de_1}{dt} + \frac{(R + r)}{L}.e_1 = 0$$

Exercise 2

3. **Applying the law of addition of voltages, show that at $t_0 = 0$ the induced e.m.f. $e_1 = -E$**

$$u_{AB} = u_{AM} + u_{MB}$$

$$E = Ri + (ri - e_1)$$

$$E = (R + r)i - e_1$$

At $t_0 = 0, i = 0$ then: $$E = (R + r)(0) - e_1$$

$$-E = e_1$$

Exercise 2

4. Determine the expressions of A & k so that $e_1 = Ae^{-kt}$ is a solution of the differential equation.

$$e_1 = Ae^{-kt}$$

$$\frac{de_1}{dt} = -Ake^{-kt}$$

Substitute e_1 *and* $\frac{de_1}{dt}$ *in differential equation.*

$$\frac{de_1}{dt} + \frac{(R+r)}{L} . e_1 = 0$$

$$-Ake^{-kt} + \frac{(R+r)}{L} . Ae^{-kt} = 0$$

Exercise 2

$$-Ake^{-kt} + \frac{(R+r)}{L}.Ae^{-kt} = 0$$

$$+A.e^{-kt}\left[-k + \frac{(R+r)}{L}\right] = 0$$

$$-k + \frac{(R+r)}{L} = 0$$

$$k = \frac{(R+r)}{L}$$

Exercise 2

$$e_1 = Ae^{-kt}$$

At t=0, $e_1 = -E$

$$-E = Ae^0 \Rightarrow -E = A$$

$$e_1 = -Ee^{-\frac{(R+r)}{L}t}$$

Exercise 2

5. **Deduce the role of the coil in the circuit.**

Because the value of e_1 is negative ($e_1 < 0$) then:
The coil acts as a generator in opposition

6. **Show that the expression of the current $i(t)$ can be written $i = a\left(1 - e^{-\frac{t}{\tau}}\right)$ where a & τ are constants whose expressions are to be determined.**

$$e_1 = -L\frac{di}{dt} \quad \Rightarrow \quad -\frac{e_1}{L} = \frac{di}{dt} \quad \Rightarrow \quad di = -\frac{e_1}{L}dt$$

Exercise 2

$$di = -\frac{e_1}{L}dt \quad \Rightarrow \quad i = -\int \frac{e_1}{L}dt$$

$$i = -\frac{1}{L}\int e_1\,dt$$

$$i = -\frac{1}{L}\int -Ee^{-\frac{(R+r)}{L}t}\,dt$$

$$i = \frac{E}{L}\int e^{-\frac{(R+r)}{L}t}\,dt$$

Exercise 2

$$di = -\frac{e_1}{L}dt \Rightarrow i = -\int \frac{e_1}{L}dt$$

$$i = -\frac{1}{L}\int e_1\,dt$$

$$i = -\frac{1}{L}\int -Ee^{-\frac{(R+r)}{L}t}\,dt \Rightarrow i = \frac{E}{L}\int e^{-\frac{(R+r)}{L}t}\,dt$$

$$i = \frac{E}{L}\left[-\frac{1}{\frac{(R+r)}{L}}\right]e^{-\frac{(R+r)}{L}t} + C$$

Exercise 2

$$i = \frac{E}{L}\left[-\frac{1}{\frac{(R+r)}{L}}\right]e^{-\frac{(R+r)}{L}t} + C$$

$$i = -\frac{E}{L}\left[\frac{L}{R+r}\right]e^{-\frac{(R+r)}{L}t} + C \quad \Rightarrow \quad i = -\frac{E}{R+r}e^{-\frac{(R+r)}{L}t} + C$$

At $t = 0; i = 0$

$$0 = -\frac{E}{R+r}e^{0} + C \quad \Rightarrow \quad C = \frac{E}{R+r}$$

Exercise 2

$$i = -\frac{E}{R+r}e^{-\frac{(R+r)}{L}t} + C$$

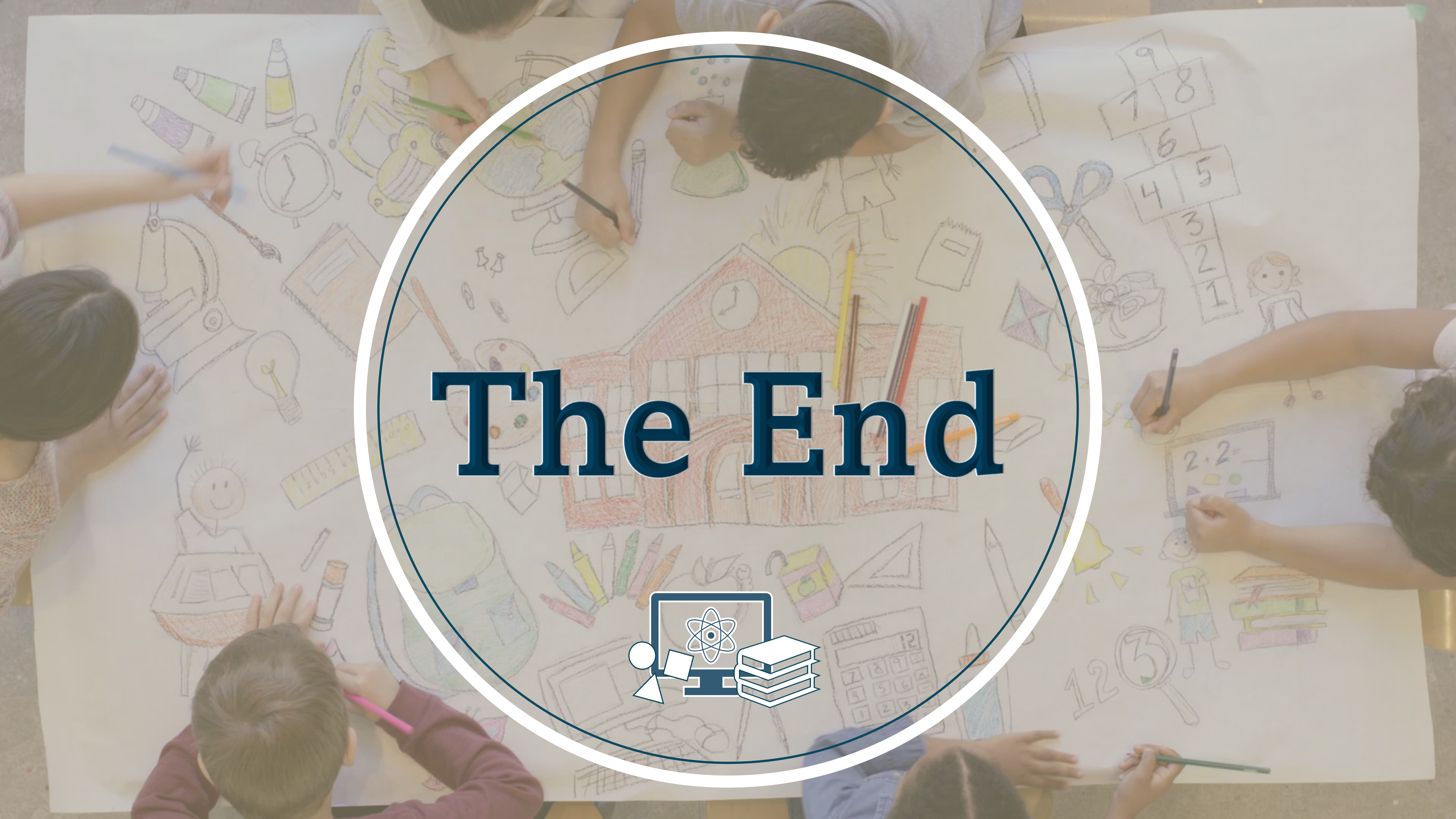
And

$$C = \frac{E}{R+r}$$

$$i = -\frac{E}{R+r}e^{-\frac{(R+r)}{L}t} + \frac{E}{R+r}$$

$$i = \frac{E}{R+r}\left(1 - e^{-\frac{(R+r)}{L}t}\right)$$

$$i = a\left(1 - e^{-\frac{t}{\tau}}\right)$$



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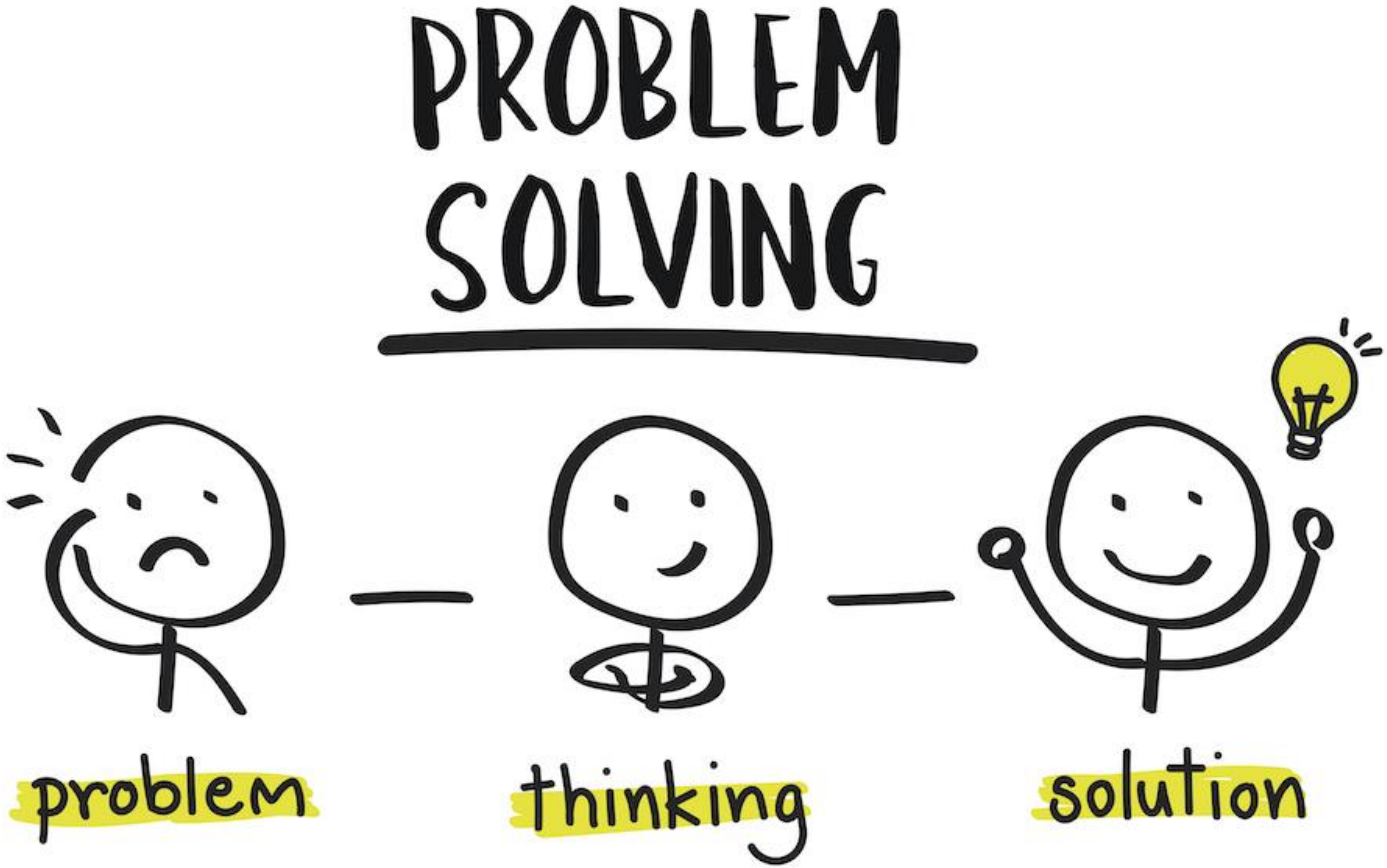
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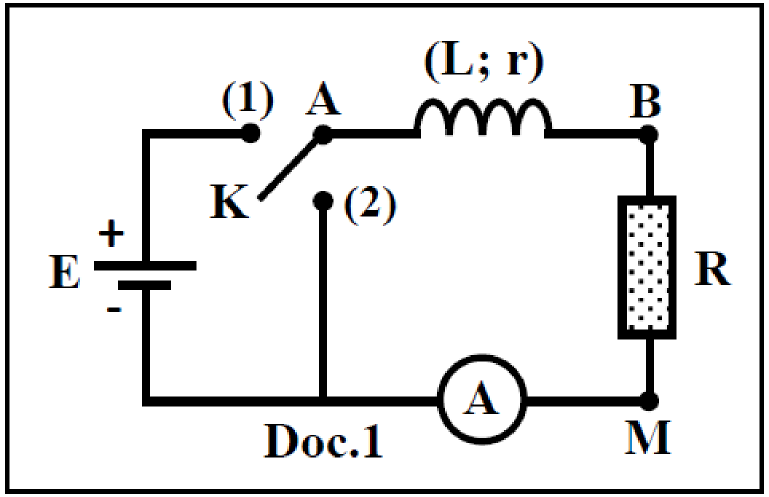
Exercise 3:

The circuit of document 1 consists of:

- An ideal generator of constant electromotive force E.
- A coil of inductance L and resistance r.
- A resistor of resistance R = 110Ω.
- A double switch K.; an ammeter and connecting wires.

The aim of this exercise is to determine the characteristics L and r of the coil.

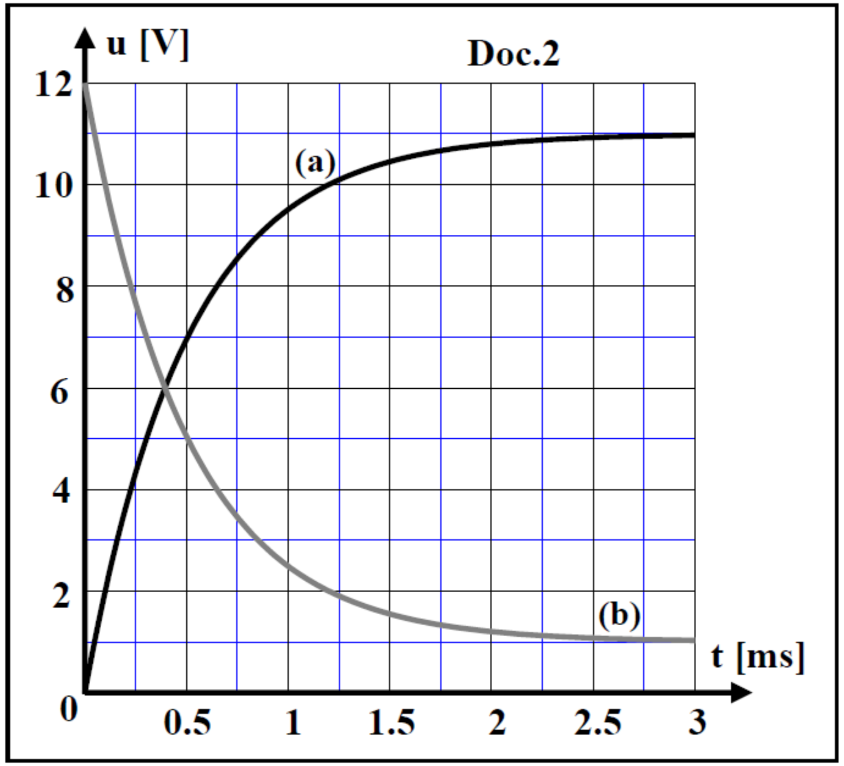


A. Analytical study of the growth of the current.

At the instant $t_0 = 0$, the switch K is turned to position (1). At the instant t, the circuit carries an electric current i.

A convenient apparatus records the variations of the voltage $u_L = u_{AB}$ across the coil and the voltage $u_R = u_{BM}$ across the resistor. We obtain the waveforms of document 2.

1. Curve (a) represents the variation of u_R as a function of time. Why?
2. Applying the law of addition of voltages, derive the first order differential equation that governs the variation of the current i as a function of time.



3. Deduce the expression of the steady state current I_0 in terms of E, R and r.
4. The solution of the differential equation has the form $i = A + Be^{-\frac{t}{\tau}}$, where A, B and τ are constants. Determine the expressions of A, B and τ in terms of E, L, R and r.

5. Determine:

   5.1. the values of I_0, r and E.

   5.2. the time constant τ; then, deduce the value of L.

B. Analytical study of the decay of current

At a new origin of time t_0, the switch K is turned to position (2). At the instant t, the circuit carries an electric current i.

1. Draw the circuit and indicate the direction of the electric current.
2. Show that the differential equation that governs the variation of the voltage u_R across the resistor is given by $\frac{du_R}{dt} + \frac{R+r}{L} u_R = 0$
3. The solution of the differential equation has the form $u_R = De^{-\alpha t}$, where D and α are constants. Show that $D = RI_0$ and $\alpha = \frac{1}{\tau}$
4. Deduce that $i = I_0 e^{-\frac{t}{\tau}}$
5. Determine the magnetic energy W_m lost by the coil between $t_0 = 0s$ and $t_1 = \tau$.

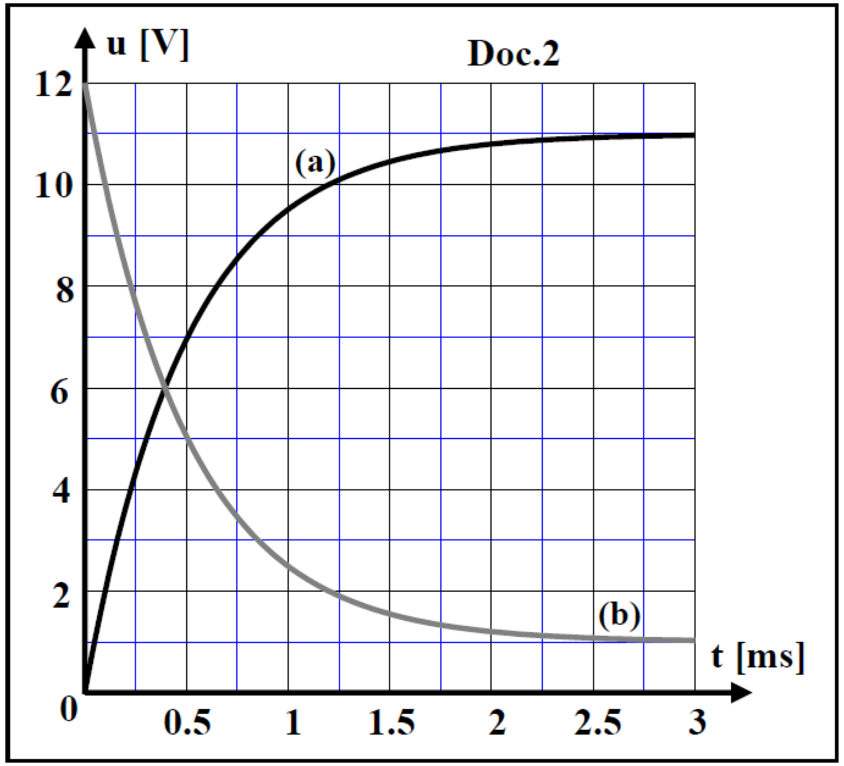
6. The energy dissipated due to joule's effect in the resistor between t_0 and t_1, is given by $W_h = \int_0^{t_1} Ri^2\,dt$.

6.1. Determine the value of W_h

6.2 Deduce the energy dissipated in the coil between t_0 and t_1.

1. Curve (a) represents the variation of u_R as a function of time. Why?

Curve (a) corresponds to u_R since it increases exponentially with time.



2. Applying the law of addition of voltages, derive the first order differential equation that governs the variation of the current i as a function of time.

Law of addition of voltages:

$$u_{AM} = u_{AB} + u_{BM}$$

$$u_G = u_R + u_L \Rightarrow E = Ri + ri + L\frac{di}{dt}$$

$$E = (R + r)i + L\frac{di}{dt}$$

3. Deduce the expression of the steady state current I_0 in terms of E, R and r.

In the steady state: $i = I_0 = constant$ and $\frac{di}{dt} = 0$

$$\Rightarrow E = (R+r)I_0 + 0 \Rightarrow I_0 = \frac{E}{}$$

4. The solution of the differential equation has the form $i = A + Be^{-\frac{t}{\tau}}$ **where A, B and τ are constants. Determine the expressions of A, B and τ in terms of E, L, R and r.**

$$i = A + Be^{-\frac{t}{\tau}} \Rightarrow \frac{di}{dt} = B(-\frac{1}{\tau})e^{-\frac{t}{\tau}} \Rightarrow \frac{di}{dt} = -\frac{B}{\tau}e^{-\frac{t}{\tau}}$$

Replace i and di/dt in the differential equation: $\Rightarrow E = (R+r)(A + Be^{-\frac{t}{\tau}}) + L(-\frac{B}{\tau}e^{-\frac{t}{\tau}})$

$$\Rightarrow E = (R+r)A + (R+r)Be^{-\frac{t}{\tau}} - L(\frac{B}{\tau}e^{-\frac{t}{\tau}}) \Rightarrow E + L(\frac{B}{\tau}e^{-\frac{t}{\tau}}) = (R+r)A + (R+r)Be^{-\frac{t}{\tau}}$$

By identification:

$$E = (R+r)A \Rightarrow A = \frac{E}{R+r}$$

$$and\ L\frac{B}{\tau} = (R+r)B \Rightarrow \tau = \frac{L}{R+r}$$

5. **Determine:**

5.1. the values of I_0, r and E.

5.2. the time constant τ; then, deduce the value of L.

5.1. In the steady state:

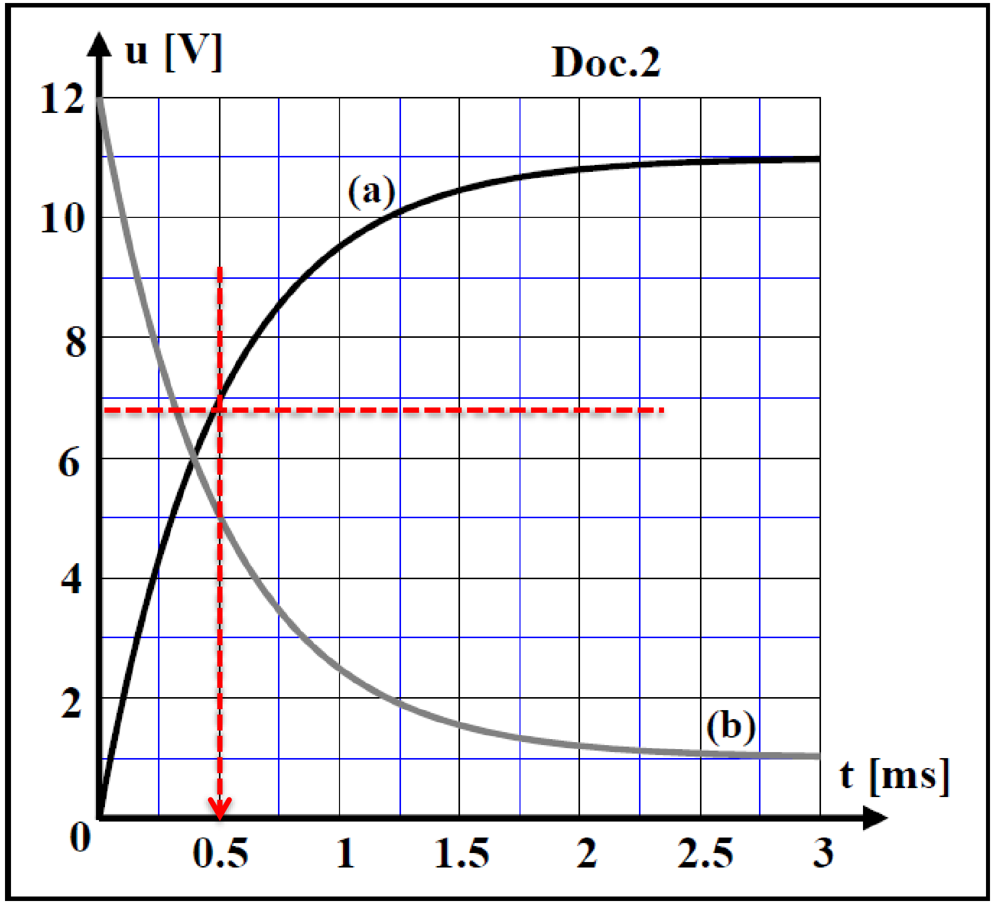
$$From\ graph: u_R = 11V\ \ and\ \ u_L = 1V$$

$$u_R = RI_0 \Rightarrow I_0 = \frac{u_R}{R} \Rightarrow I_0 = \frac{11}{110} = 0.1A$$

$$u_L = rI_0 \Rightarrow r = \frac{u_L}{I_0} \Rightarrow r = \frac{1}{0.1} = 10\Omega$$

$$I_0 = \frac{E}{R+r} \Rightarrow E = (R+r)I_0$$

$$\Rightarrow E = (110+10)\times 0.1 = 12V$$



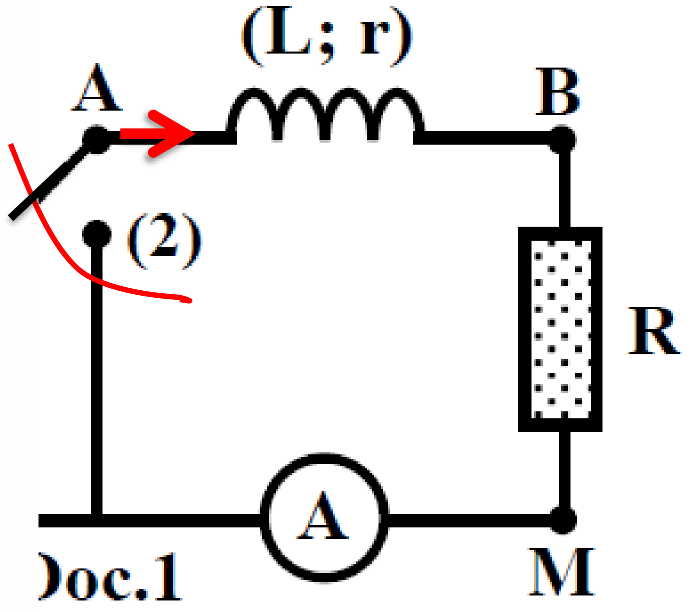
5.2. At t = τ; u_R=0.63u_{Rmax} = 0.63×11=6.93V.

Graphically, τ=0.5ms.

$$But\ \tau = \frac{L}{R+r} \quad \Rightarrow L = (R+r)\tau = (110+10)\times 0.5\times 10^{-3} \quad \Rightarrow L = 0.06H$$

1. **Draw the circuit and indicate the direction of the electric current.**

2. **Show that the differential equation that governs the variation of the voltage u_R across the resistor is given by** $\frac{du_R}{dt} + \frac{R+r}{L} u_R = 0$



Apply law of addition of voltages:

$u_L + u_R = 0 \Rightarrow ri + L\frac{di}{dt} + Ri = 0$

$\Rightarrow (R+r)i + L\frac{di}{dt} = 0$

$But\ u_R = Ri \Rightarrow i = \frac{u_R}{R}$

$\Rightarrow (R+r)\frac{u_R}{R} + L\frac{d(\frac{u_R}{R})}{dt} = 0 \quad \Rightarrow (R+r)\frac{u_R}{R} + L\frac{du_R}{Rdt} = 0$

$Multiply\ by: \frac{R}{L} \quad \Rightarrow \frac{du_R}{dt} + \frac{(R+r)}{L}u_R = 0$

3. The solution of the differential equation has the form $u_R = De^{-\alpha t}$ where D and α are constants. Show that $D=RI_0$ and $\alpha = \frac{1}{\tau}$

$$u_R = De^{-\alpha t} \Rightarrow \frac{du_R}{dt} = D(-\alpha)e^{-\alpha t} = -D\alpha e^{-\alpha t}$$

Replace u_R and du_R/dt in the differential equation:

$$\Rightarrow -D\alpha e^{-\alpha t} + \frac{(R+r)}{L}(De^{-\alpha t}) = 0$$

$$\Rightarrow De^{-\alpha t}(-\alpha + \frac{(R+r)}{L}) = 0$$

$$But\ De^{-\alpha t} \neq 0 \quad \Rightarrow (-\alpha + \frac{(R+r)}{L}) = 0 \Rightarrow \alpha = \frac{(R+r)}{L}) = \frac{1}{\tau}$$

$$At\ t_0 = 0, i = I_0\ and\ u_R = RI_0 \Rightarrow RI_0 = De^0 \Rightarrow D = RI_0$$

4. **Deduce that** $i = I_0 e^{-\frac{t}{\tau}}$

$$u_R = De^{-\alpha t} \Rightarrow Ri = De^{-\alpha t} \Rightarrow i = \frac{De^{-\alpha t}}{R} \Rightarrow i = \frac{RI_0 e^{-\frac{1}{\tau}t}}{R} \Rightarrow i = I_0 e^{-\frac{t}{\tau}}$$

5. **Determine the magnetic energy** Wm **lost by the coil between** $t_0 = 0s$ **and** $t_1 = \tau$**.**

$$At\, t = 0 : i_0 = I_0 e^0 = 0.1A$$

$$At\ t_1 = \tau :\ i_1 = I_0 e^{-1} = 0.1 \times 0.37 = 0.037A$$

$$Magnetic\ enery\ lost : W_m = W_{t=0} - W_{t=\tau} \Rightarrow W_m = \frac{1}{2}Li_0^2 - \frac{1}{2}Li_1^2$$

$$\Rightarrow W_m = \frac{1}{2}L(i_0^2 - i_1^2) \Rightarrow W_m = \frac{1}{2}0.06(0.1^2 - 0.037^2) \Rightarrow W_m = 2.59 \times 10^{-4}\ J$$

6. The energy dissipated due to joule's effect in the resistor between t0 and t1, is given by $W_h = \int_0^{t_1} Ri^2 dt$

6.1. Determine the value of W_h.

6.2 Deduce the energy dissipated in the coil between t_0 and t_1.

$$6.1. W_h = \int_0^{t_1} Ri^2 dt \quad \Rightarrow W_h = \int_0^{t_1} R(I_0 e^{-\frac{t}{\tau}})^2 dt$$

$$\Rightarrow W_h = \int_0^{t_1} RI_0^2 e^{-\frac{2t}{\tau}} dt \quad \Rightarrow W_h = RI_0^2 \int_0^{t_1} e^{-\frac{2t}{\tau}} dt$$

$$\Rightarrow W_h = RI_0^2 \{[\frac{1}{\frac{-2}{\tau}}] e^{-\frac{2t}{\tau}} \Big|_0^{t_1 = \tau} \quad \Rightarrow W_h = -\frac{RI_0^2 \tau}{2} e^{-\frac{2t}{\tau}} \Big|_0^{t_1 = \tau} \quad \Rightarrow W_h = -\frac{RI_0^2 \tau}{2} (e^{-2} - e^0)$$

$$\Rightarrow W_h = -\frac{110 \times 0.1^2 \times 0.5 \times 10^{-3}}{2} (e^{-2} - 1) \quad \Rightarrow W_h = 2.38 \times 10^{-4} J$$

6.2. $W = W_m - W_h$

$$\Rightarrow W = 2.59 \times 10^{-4} - 2.38 \times 10^{-4}$$

$$\Rightarrow W = 0.21 \times 10^{-4}\ J$$